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The effects of transpiration on the flow and heat transfer over a moving permeable surface in a parallel stream

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ABSTRACT

The steady boundary layer flow over a moving permeable sheet in a viscous and incompressible fluid is considered. In addition to the mass transfer from the plate (suction or injection), the viscous dissipation term is also included into the energy equation. The sheet is assumed to move in the same or opposite direction to the free stream. The governing partial differential equations are first transformed into ordinary differential equations before they are solved numerically by a finite-difference scheme. The numerical results are compared with known results from the open literature for some particular cases of the present study, to support their validity. The effects of the governing parameters on the flow and thermal fields are examined. The numerical results indicate that dual solutions exist when the sheet and the free stream move in the opposite directions. Moreover, compared to injection, suction increases the skin friction coefficient and the heat transfer rates at the surface, besides delays the boundary layer separation.

1. Introduction

The behavior of boundary layer flow due to a moving flat surface immersed in an otherwise quiescent fluid was first studied by Sakiadis [1], who investigated it theoretically by both exact and approximate methods. Significant differences were found between this behavior and the behavior of the boundary layer on a stationary surface in a moving fluid considered by Blasius [2]. The skin friction is about 30% higher for the Sakiadis result compared to the Blasius result. Tsou et al. [3] showed that the Sakiadis flow is physically realizable under laboratory conditions, and they determined the heat transfer rates for certain values of the Prandtl number.

Following the Blasius and Sakiadis works, Abdelhafez [4] investigated the boundary layer flow on a moving flat surface in a parallel stream, but only the case when the surface and the free stream move in the same direction was considered. He showed that the classical Blasius and Sakiadis problems are two special cases of his problem. Further, Chappidi and Gunnerson [5] considered the similar problem, and reported a closed form analytical solution as a function of the velocity difference between the surface and the free stream. Afzal et al. [6] formulated a single set of boundary conditions, employing a composite velocity $U_w + U_\infty$, where U_w is the surface velocity and U_∞ is the free stream velocity, instead of considering U_w and U_∞ separately as done by Abdelhafez [4], and Chappidi and Gunnerson [5]. Moreover, Afzal et al. [6] also considered the case when the surface and the free stream move in the opposite directions, and found that dual solutions exist for this case. The existence and nonuniqueness solutions to the boundary layer equations when a surface moves in opposite direction to the free stream was discussed also by Weidman et al. [7], Hussaini and Lakin [8], and the rigorous proof is given by Hussaini et al. [9]. Very recently, Cortell [10] extended the work of Afzal et al. [6], by taking into account the effects of viscous dissipation on the thermal field. However, the existence of dual solutions was not reported, owing to the smaller range of velocity ratio between the free stream and the composite velocity considered in that study. The objective of the present paper is, therefore, to show that dual solutions exist when the velocity ratio exceeds unity, i.e. the sheet moves in opposite direction to the free stream, besides investigates the effects of suction and injection on the flow and thermal fields.

The study of boundary layer behavior over a moving surface in a parallel stream has important practical applications such as the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyors, the boundary layer along a liquid film in condensation processes, paper production, etc. (see Chappidi and Gunnerson [5] and Cortell [10]). The process of suction and injection (blowing) has also its importance in many engineering applications

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such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surfaces, prevent corrosion or scaling and reduce the drag (see Labropulu et al. [11]).

2. Flow analysis

Consider the steady two-dimensional laminar flow over a moving permeable sheet with constant velocity U_w, in the same or opposite direction to the free stream U_∞. The x-axis extends parallel to the sheet, while the y-axis extends upwards, normal to the surface of the sheet. The boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where u and v are the velocity components in the x and y directions, respectively, and ν is the kinematic viscosity. We shall solve Eqs. (1) and (2) subject to the following boundary conditions:

$$u = U_w,\ v = V_w \quad \text{at} \quad y = 0, \qquad u \to U_\infty \quad \text{as} \quad y \to \infty, \tag{3}$$

where V_w is the mass transfer velocity at the surface of the sheet with $V_w > 0$ for injection (blowing), $V_w < 0$ for suction and $V_w = 0$ corresponds to an impermeable sheet.

In order to solve Eqs. (1)–(3), we introduce the following similarity transformation (see Afzal et al. [6] and Hussaini et al. [9]):

$$\psi = (2\nu x U)^{1/2} f(\eta), \quad \eta = \left(\frac{U}{2\nu x}\right)^{1/2} y, \tag{4}$$

where $U = U_w + U_\infty$, η is the similarity variable, f is the dimensionless stream function and ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which identically satisfy Eq. (1). The composite velocity $U = U_w + U_\infty$ was first introduced by Afzal et al. [6] to enable the formulation of a single set of equation, and not to consider two cases separately when $U_w > U_\infty$ and $U_w < U_\infty$ as analysed by Abdelhafez [4] and Chappidi and Gunnerson [5]. Using (4) we obtain

$$u = Uf'(\eta), \quad v = \left(\frac{\nu U}{2x}\right)^{1/2}(\eta f' - f), \tag{5}$$

where primes denote differentiation with respect to η. In order that similarity solutions of Eqs. (1)–(3) exist, we take

$$V_w(x) = -\left(\frac{\nu U}{2x}\right)^{1/2} f_0, \tag{6}$$

where $f_0 = f(0)$ is a non-dimensional constant which determines the transpiration rate at the surface, with $f_0 > 0$ for suction, $f_0 < 0$ for injection, and $f_0 = 0$ corresponds to an impermeable sheet. Using (4) and (5), Eq. (2) reduces to the similarity equation

$$f''' + ff'' = 0. \tag{7}$$

The boundary conditions (3) now become

$$f(0) = f_0, \quad f'(0) = 1 - r, \quad f'(\infty) \to r, \tag{8}$$

where r is the velocity ratio parameter defined as (see Afzal et al. [6] and Cortell [10])

$$r = \frac{U_\infty}{U} \tag{9}$$

with $0 < r < 1$ corresponding to the sheet moving in the same direction to the free stream, while $r < 0$ and $r > 1$ are when they move in the opposite directions. We notice that when $f_0 = 0$ (impermeable sheet), Eqs. (7) and (8) reduce to those of Afzal et al. [6] and Cortell [10], while the cases $r = 1$ and $r = 0$ correspond to the classical Blasius [2] and Sakiadis [1] problems, respectively.

The physical quantity of interest is the skin friction coefficient C_f which can be shown to be given by

$$\frac{1}{2} C_f \sqrt{Re_x} = \frac{1}{\sqrt{2}} f''(0), \tag{10}$$

where $Re_x = Ux/\nu$ is the local Reynolds number.

Table 1 shows the comparison of the numerical values of the present study with those obtained by Cortell [10], which are in very good agreement. The new result in this table is the existence of dual solutions when $r > 1$. This result was not reported in [10], owing to the smaller range of r considered in that paper, i.e. $0 \le r \le 1$. Further, for impermeable sheet ($f_0 = 0$), the values of $f''(0)$ presented in Table 2 can be reduced to those of Blasius [2] when $r = 1$, and those of Sakiadis [1] when $r = 0$ if the factor "2" in Eq. (4) is neglected, to give $f''(0)/\sqrt{2} = 0.332$ and $f''(0)/\sqrt{2} = -0.44375$, respectively.

Fig. 1 presents the variation of the skin friction coefficient $f''(0)$ with r for $f_0 = -0.1$, 0 and 0.2. It is seen from this figure that the value of $f''(0)$ is positive when $r > 0.5$, zero when $r = 0.5$ and negative when $r < 0.5$. Physically, positive value of $f''(0)$ means the fluid exerts a drag force on the sheet, while negative value means the opposite. There is no skin friction when $r = 0.5$, since for this

Table 1
The velocity gradient at the surface $f''(0)$ for various values of r when $f_0 = 0$

r	Cortell [10]	Present results	
		Upper branch	Lower branch
0	−0.627547	−0.627562	
0.1	−0.493711	−0.493760	
0.2	−0.363308	−0.363346	
0.3	−0.237132	−0.237133	
0.4	−0.115777	−0.115810	
0.5	0.0	0.0	
0.6	0.109652	0.109638	
0.7	0.212374	0.212373	
0.8	0.307378	0.307355	
0.9	0.393567	0.393563	
1.0	0.469602	0.469601	
1.1		0.533708	0.001493
1.2		0.583178	0.016171
1.3		0.613646	0.051941
1.4		0.616140	0.117886
1.5		0.565821	0.241872

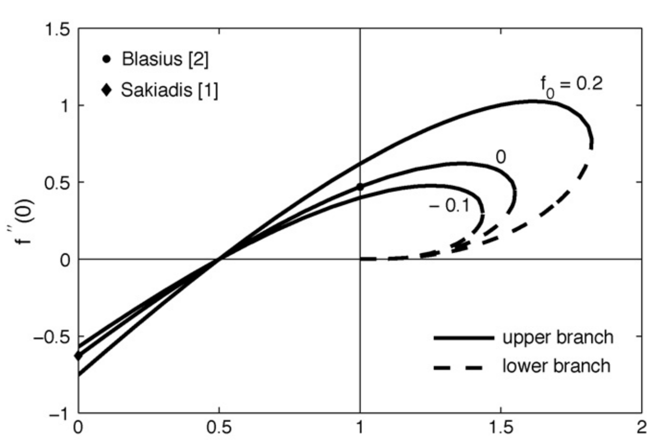

Fig. 1. Skin friction coefficient $f''(0)$ as a function of r when $f_0 = -0.1, 0, 0.2$.

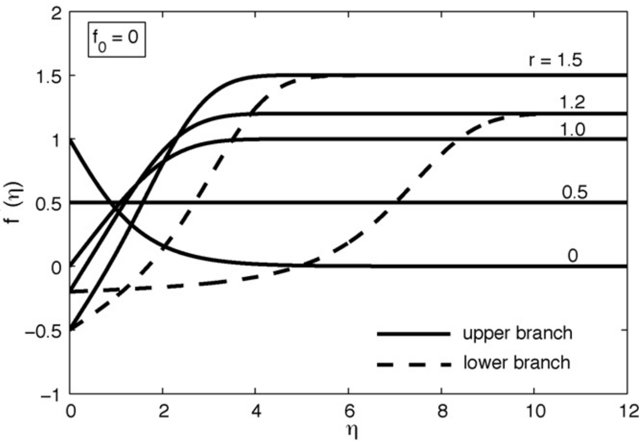

Fig. 2. Velocity profiles $f'(\eta)$ for various values of r when $f_0 = 0$ (impermeable sheet).

case, the sheet and the fluid move with the same velocity. The solution of Eq. (7) subjected to (8) is unique when $r \le 1$, while dual solutions are found to exist when $r > 1$, which is in agreement with the results reported by Afzal et al. [6]. For a particular value of f_0, the solution could be obtained up to the critical value of r, say r_c, where the upper branch solution meets the lower branch solution. Beyond this critical value, no solution is obtained since the boundary layer has separated from the surface. Based on our computations, $r_c = 1.436$, 1.548 and 1.821 for $f_0 = -0.1$, 0 and 0.2, respectively. This value of r_c when $f_0 = 0$ (impermeable sheet) is in excellent agreement with those reported by Afzal et al. [6].

For dual solutions, the upper branch is likely to be stable and physically relevant solution, since it has smaller boundary layer thickness (see Figs. 2 and 3), and the only solution for the case $r \le 1$. For this solution, the magnitude of $f''(0)$ is higher for suction ($f_0 > 0$) compared to injection ($f_0 < 0$). Further, suction increases the range of r for which the solution exists, whereas injection decreases it. Thus, suction delays the boundary layer separation, whereas injection accelerates it.

The selected velocity profiles are presented in Figs. 2 and 3 for fixed values of f_0 and r, respectively. These profiles show that the boundary conditions (8) are satisfied, which support the validity of the numerical results obtained. Figs. 2 and 3 also show the existence of two different velocity profiles, both satisfy the boundary conditions (8), for a particular value of r and f_0, respectively. These profiles support the dual nature of $f''(0)$ as presented in Table 1 as well as Fig. 1, for $1 < r < r_c$.

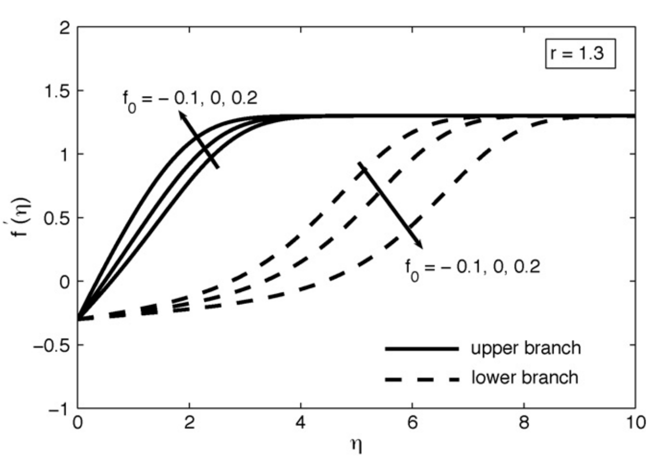

Fig. 3. Velocity profiles $f'(\eta)$ for various values of f_0 when $r = 1.3$.

3. Heat transfer analysis

Under the boundary layer approximations, and taking into account the viscous dissipation effects, the energy equation is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{11}$$

where T is the temperature inside the boundary layer, α is the thermal diffusivity and c_p is the specific heat at constant pressure. We assume that the boundary conditions of Eq. (11) are given by

$$T = T_w \quad \text{at} \quad y = 0; \quad T \to T_\infty \quad \text{as} \quad y \to \infty, \tag{12}$$

where T_w and T_∞ are constants, with $T_w > T_\infty$. By defining the dimensionless temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{13}$$

and by using relations (4), Eqs. (11) and (12) become

$$\frac{1}{Pr}\theta'' + f\theta' + Ec(f'')^2 = 0, \tag{14}$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0, \tag{15}$$

where $Pr = \nu/\alpha$ is the Prandtl number and $Ec = U^2/c_p(T_w - T_\infty)$ is the Eckert number.

The local Nusselt number (rate of heat transfer at the surface) Nu_x is derived using (13), and is given by

$$\frac{Nu_x}{\sqrt{Re_x}} = -\frac{1}{\sqrt{2}}\theta'(0). \tag{16}$$

Table 2 presents the values of $-\theta'(0)$ for some values of Ec and r when $f_0 = 0$ (impermeable sheet). The present results are very well comparable with those results reported by Cortell [10]. Dual solutions are found to exist when $r > 1$, a new result that was not reported in [10]. The variation of $-\theta'(0)$ with r for $f_0 = -0.1$, 0 and 0.2 when $Ec = 0.03$ and $Pr = 0.7$ is depicted in Fig. 4, with the selected temperature profiles are given in Figs. 5–7. For the upper branch solution (likely to be physically relevant and stable solution), the heat transfer rate at the surface is higher for suction compared to injection. This is because suction increases the surface shear stress, while injection decreases it. Fig. 4 also shows that there is heat transfer from the sheet to the fluid ($-\theta'(0) > 0$) when $r = 0.5$, even though there is no skin friction for this case (see Fig. 1), since the sheet and the fluid are at different temperatures.

Figs. 5–7 show the existence of two different temperature profiles for a particular value of r, f_0 and Pr, respectively, which produce two different values of $-\theta'(0)$ as shown in Fig. 4 . It is seen from these figures that the boundary layer thickness is smaller for the upper branch solution compared to the lower branch solution. Also, Figs. 6 and 7 show that for the upper branch solution, the temperature

Table 2
The values of $-\theta'(0)$ for various values of Ec and r when $f_0 = 0$ and $Pr = 0.7$ (air)

Ec	r	Cortell [10]	Present results	
			Upper branch	Lower branch
0.0	0.1	0.493622	0.493641	
0.03		0.489284	0.489305	
	0.3	0.484773	0.484789	
	0.7	0.452177	0.452195	
	0.9	0.425217	0.425233	
	1.2		0.368048	0.023648
	1.5		0.264915	0.133918
0.1	1.2		0.346889	0.022557
	1.5		0.234991	0.119907

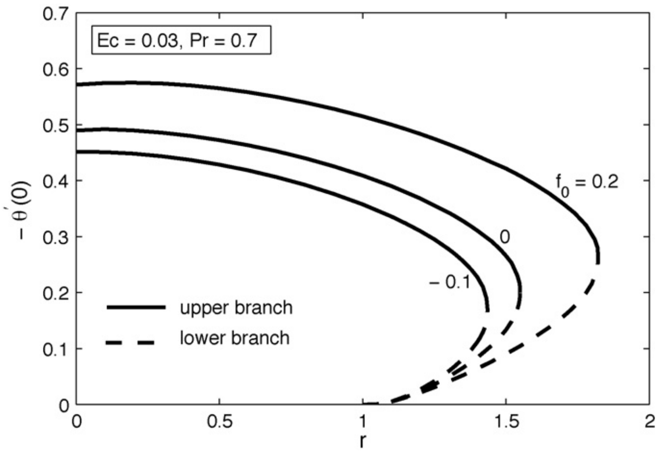

Fig. 4. Local Nusselt number $-\theta'(0)$ as a function of r for $f_0 = -0.1$, 0, 0.2 when $Ec = 0.03$ and $Pr = 0.7$.

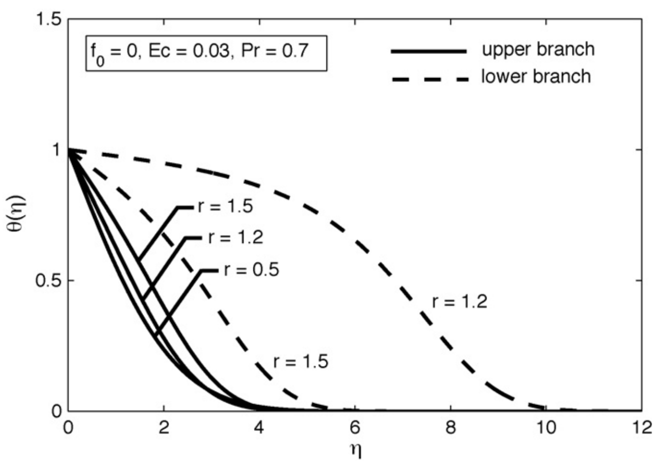

Fig. 5. Temperature profiles $\theta(\eta)$ for various values of r when $f_0 = 0$, $Ec = 0.03$ and $Pr = 0.7$ (air).

gradient at the surface is higher for higher values of f_0 and Pr, and this implies increasing manner of the local Nusselt number with increasing values of these parameters. Thus, the heat transfer rate at the surface is higher for higher values of f_0 and Pr.

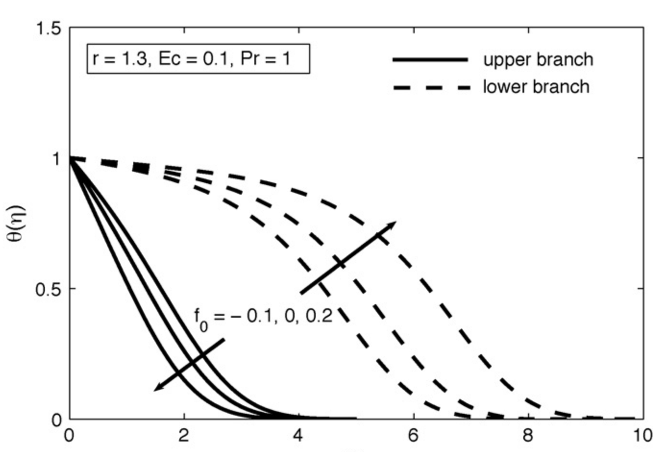

Fig. 6. Temperature profiles $\theta(\eta)$ for various values of f_0 when $r = 1.3$, $Ec = 0.1$ and $Pr = 1$.

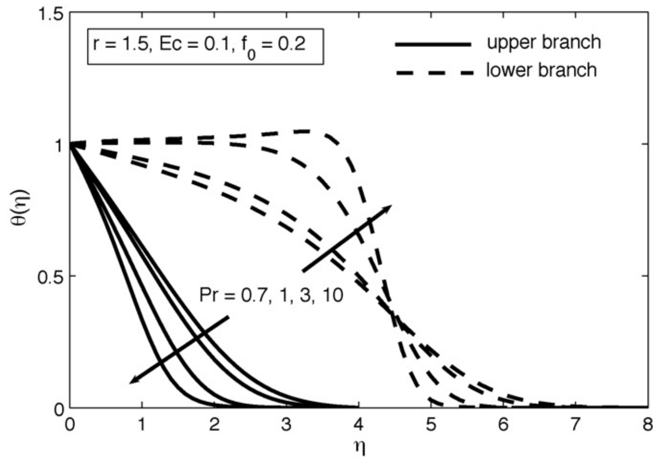

Fig. 7. Temperature profiles $\theta(\eta)$ for various values of Pr when $r = 1.5$, $Ec = 0.1$ and $f_0 = 0.2$.

4. Conclusions

We studied theoretically the problem of steady boundary layer flow and heat transfer over a moving sheet in a parallel stream. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation, which are more convenient for numerical computation. The transformed nonlinear ordinary differential equations are then solved numerically by a finite-difference scheme known as the Keller box method described in [12]. The numerical results obtained are then compared with previously reported cases available from the open literature, and they are found to be in very good agreement. From the present investigation, we found that dual solutions exist when the motion of the sheet is in opposite direction to the free stream, the case that was not considered in [10]. The upper branch is likely to be physical and stable solution, and for this solution, suction increases the magnitude of the skin friction coefficient and in consequence increases the heat transfer rate at the surface, while injection shows the opposite manner. Moreover, suction delays the boundary layer separation, while injection (blowing) accelerates it.

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